Mean Motion Resonances in the Main Rings of Saturn

This document briefly describes the set of tables of mean motion resonances in the rings of Saturn. The tables are organized by perturbing satellite. The tables contain all mean motion resonances, up to fourth order (up to second order resonances for Pan and Daphnis). The tables are sorted by resonance order (lowest order resonances first), and then by increasing semi-major axis of the resonance.

1. Approach & Notation

Resonance locations were generated recursively. The calculations included the J_2, J_4, and J_6 of Saturn. The resonance 'strengths' given in the tables are low order terms of the Fourier series expansion of the disturbing function. They are generally restricted to fourth order in the eccentricities and inclinations, except for Pan and Daphnis which are restricted to second order terms. These 'strength' expressions are potentials, not forces or torques. For a complete discussion of the series expansion of the disturbing function and its interpretation using Lagrange's planetary equations, see Murray & Dermott, 1999.

A Fourier series expansion of the disturbing function can be written for both the interior and exterior body (generally ring particle and satellite respectively, except in the case of an embedded moonlet such as Pan). The expansion for the inner body can be written in the form:

$$R = \mu' \sum S \cos\varphi \qquad \text{where } \mu' = Gm' \tag{1}$$

Primes refer to quantities associated with the outer body.

In equation (1), S gives the strength of a particular term and is a function of the semi-major axes, eccentricities, and inclinations of the two bodies. The argument of the cosine, φ, is referred to as the argument of the disturbing function, and is a linear combination of six angles: the mean longitude, longitude of pericenter, and longitude of the ascending node ($\lambda, \tilde{\omega},$ and Ω respectively) for each body.

$$\varphi = j_1\lambda' + j_2\lambda + j_3\tilde{\omega}' + j_4\tilde{\omega} + j_5\Omega' + j_6\Omega \tag{2}$$

where the j_i are integers, referred to as coefficients of the argument,

$$\text{and} \quad \sum j_i = 0 \tag{3}$$

The order of the resonance is the absolute value of the sum of j_1 and j_2.

Each strength term, S, is the product of two functions,

$$S = f_i(\alpha) g_i(e', e, s', s) \tag{4}$$

where α is the ratio of the semi-major axis and is less than 1, and s is sine of one half the inclination

$$s = \sin(1/2\, I)\,. \tag{5}$$

In the resonance tables, the values for the f_i's are given explicitly while the g_i's are given in terms of the orbital elements. Generally, the lowest order term for a given argument will have

$$g_i = (e'^{j_3} e^{j_4} s'^{j_5} s^{j_6}) \tag{6}$$

2. Information in the Tables

Input Parameters

A variety of sources were used for the input parameters. All of the parameters and their sources are given in Section 3 below. Full citations of the source documents are included in the accompanying ref.cat file.

File Naming Conventions

Filenames are in the form nnn_aaa.tab where

nnn is the NAIF ID of the perturbing satellite,

aaa indicates one or more of

- epoch (vgr, hst, or cas),
- configuration (Janus interior (jinside) or exterior (joutside) to Epimetheus),
- relative location (extper) for resonances interior to the orbit of Pan or Daphnis (intper) for resonances exterior to the orbit of Pan or Daphnis. For the 'intper' files, the primes in the previous discuss refer to the resonance location rather than the perturbing satellite, and the expression for the disturbing function is different than the one given in equation (1). See Murray and Dermott, 1999 for details of the alternate expression. The last example in the next section highlights the differences in the tables for interior vs. exterior perturber.

Columns in the Resonance Tables

The tables are PDS compliant, fixed width, comma separated variable files. Each table contains six columns:

NAIF ID, "{j_1, j_2, j_3, j_4, j_5, j_6}", a, n, k, "{strength}"

1. NAIF ID of the perturber
2. Coefficients of the argument of the resonance, in the form {j_1, j_2, j_3, j_4, j_5, j_6}.
3. The semi-major axis of the resonance.
4. The average mean motion of a particle at the resonance location.
5. The order of the resonance.
6. The lowest order terms of the strength expression from the disturbing function. The expression may include multiple terms each of which will have some combination of products of the eccentricities (e) and inclinations (s = *sin(½ I)*) of the resonance (r) location and/or perturbing satellite (s). Hence the four possible factors are er, es, sr, and ss. Any or all may be raised to a power.

Examples:

- A third order Janus 11:8 resonance involving the inclination at the resonance and the eccentricity and inclination of the perturbing satellite:

610,"{ 11, -8,-1, 0,-1,-1}",122512.9, 713.118, 3,"{-7.320E-05*es*sr*ss}"

The strength expression is $\left(-7.320*10^{-5}\right)*es*sr*ss$, and es is the eccentricity of the satellite, sr is sin(½ I) at the resonance, and ss is sin(½ I) of the perturbing satellite.

- The Janus fourth order 8:4 horizontal Lindblad resonance:

610,"{   8, -4,-3,-1, 0, 0}", 95861.6,1032.313, 4,"{-6.260E-05*er*es^3}   }"

The strength expression is $\left(-6.260*10^{-5}\right)*er*es^{3}$. Note the correspondence between the coefficients of the argument and the exponents on the orbital elements. The absolute value of the coefficient of the longitude of pericenter of the satellite (-3) is the exponent on the satellite's eccentricity, and the absolute value of the coefficient of the longitude of pericenter of the resonance location is the exponent on the resonance location eccentricity. This pattern is consistent for the lowest order term in each of the strength expressions.

- Internal vs. external perturber. The Pan 800:799 inner and outer horizontal Lindblad resonances:

  extper: 618,"{ 800,-799, 0,-1, 0, 0}",133472.8, 626.811, 1,"{-1.588E-06*er}
  intper: 618,"{ 800,-799,-1, 0, 0, 0}",133693.7, 625.253, 1,"{ 1.616E-06*er}

  Recall that in our notation j_1, j_3, and j_5 are associated with the external body. Consequently for resonance locations interior to the orbit of the perturbing satellite (extper), j_1, j_3, and j_5 are associated with the perturbing satellite (always true for every perturber except Pan and Daphnis), but for the resonances in the rings beyond the orbit of the perturber (intper) j_1, j_3, and j_5 are associated with the resonance location.

3. Input parameters and NAIF IDs

Superscripts on the satellite names in the following tables identify the PDS reference ID (REFID) for the source document. Each REFID is associated with a complete citation in the accompanying ref.cat file.

Superscript	REFID
a	EVANS2001
b	FRENCHETAL2003
c	GIORGINIETAL1996
d	JACOBSONETAL2006
e	JACOBSONETAL2006B
f	JACOBSONETAL2008
g	PORCO2007

Table 1. Key to References

R	km	60330.0
GM	km^3s^{-2}	37931207.7
j_2		$1.629071*10^{-2}$
j_4		$- 9.3583*10^{-4}$
j_6		$8.614*10^{-5}$

Table 2. Saturn Parameters (all from JACOBSONETAL2006B)

Satellite NAIF IDs and Masses

		Mimas [d]	**Enceladus** [e]	**Janus** [f]	**Epimetheus** [f]
NAIF ID		601	602	610	611
m	$*10^{13}$ kg	$3.75255*10^6$	$1.08*10^7$	$1.9*10^5$	$5.26*10^4$
		Prometheus [f]	**Pandora** [f]	**Pan** [g]	**Daphnis** [g]
		616	617	618	635
m	$*10^{13}$ kg	$1.59*10^4$	$1.37*10^4$	$4.95*10^2$	8.4

Table 3. Satellite NAIF IDs and Masses

For a complete list of NAIF IDs, see the PDS NAIF Node web site: http://pds-naif.jpl.nasa.gov/

Satellite Orbital Elements

The orbital elements are grouped by 'era' – a general time frame. The values used for each satellite are those given in the reference documents, for specific epochs. Thus while elements for several satellites are given for the 'Cassini era', they are not necessarily tied to the same epoch. There are separate tables for the Janus – Epimetheus pair and for the Prometheus – Pandora pair since their orbits change on relatively short time scales. For Janus and Epimetheus we provide results for the two configurations of the satellites (Janus interior to Epimetheus and Janus exterior to Epimetheus). For Prometheus we provide results for three era's (Voyager encounters, HST – the period between the Voyager 2 and Cassini observations, and Cassini – roughly corresponding to about the first two years of the Cassini encounter). Here are the era's used and the approximate date to which each refers:

Cassini	2005/01	
HST	1998/01	Used for Prometheus and Pandora.
Voyager	1981/01	Used for Prometheus and Pandora.
Janus Interior	2005/01	Used for Janus and Epimetheus
Janus Exterior	2007/01	Used for Janus and Epimetheus

Cassini era

		Mimas [c]	Enceladus [c]	Atlas [f]	Pan [f]	Daphnis [f]
a	km	186021.8	238412.4	137670.0	133584.0	136505.5
n	deg/day	380.004677	261.903558	598.312351	626.031735	605.979162
e		0.019486892	0.004958504	0.0012	0.0000144	0.0000331
i	deg	1.572	0.009	0.0031	0.0001	0.0036

Table 4. Orbital elements for Mimas, Enceladus, Atlas, Pan, and Daphnis.

Note: average orbital elements referenced by 'c', Giorgini etal., 1996, were generated by the JPL Horizons system, http://ssd.jpl.nasa.gov/?horizons using JPL ephemeris file SAT286.

Janus – Epimetheus

		Janus Exterior		Janus Interior	
		Janus [f]	Epimetheus [f]	Janus [f]	Epimetheus [f]
a	km	151460.0	151410.0	151440.0	151490.0
n	deg/day	518.238030	518.486468	518.345648	518.097622
e		0.0068	0.0098	0.0068	0.0097
i	deg	0.164	0.3524	0.1639	0.3525

Table 5. Orbital elements for Janus and Epimetheus based on relative semi-major axes.

Prometheus – Pandora

		Voyager		HST		Cassini	
		Prometheus [a]	Pandora [a]	Prometheus [b]	Pandora [b]	Prometheus [f]	Pandora [f]
a	km	139377.3	141713.3	139377.6	141713.1	139380.0	141710.0
n	deg/day	587.28942	572.78439	587.28747	572.7856	587.285237	572.788589
e		0.00192	0.0044	0.00192	0.0045	0.0022	0.0042
i	deg	0.03	0.054	0.03	0.054	0.0075	0.0507

Table 6. Orbital elements for Prometheus and Pandora for three eras.

4. Citing the resonance tables.

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